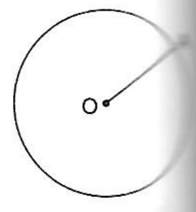
10 CIRCLES

Circle and Its Related Terms



- **Circle:** The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is called the centre of the circle and the fixed distance from the centre is called the radius of the circle. In the given figure, O is the centre and OA is the radius of the circle.

Terms related to circle:

(*i*) **Chord:** A line segment joining two point on the circle is called the chord of the circle. In fig (*i*), PQ is a chord of the circle.

(*ii*) **Diameter:** It is the longest chord of the circle, it passes through the centre of a circle and two times the radius.

AOB or AB is a diameter [fig (*i*)].

(*iii*) **Arc:** A part of a circle between two points on it is called an arc. Smaller part of the circle is called a minor arc, whereas larger part of circle is called a major arc.

In fig (*ii*), PQ is the minor arc and PRQ is the major arc.

(*iv*) **Sector:** The region between arc and two radii is called sector [fig (*iii*)].

(*v*) Minor arc corresponds to minor sector and major arc corresponds to major sector [see fig (*iv*)].

(*vi*) **Segments:** The region between a chord and either of its arcs is called segment of a circle.

A circle has two segments.

- Major segment and Minor segment [see fig (*iii*)]

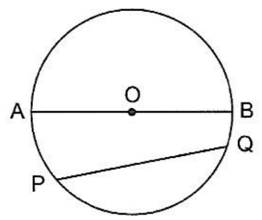


Fig. (*i*)

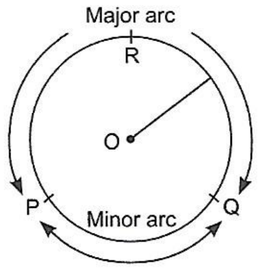


Fig. (*ii*)

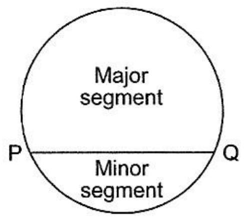


Fig. (*iii*)

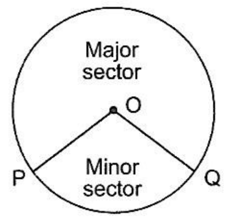


Fig. (*iv*)

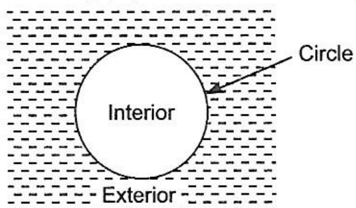
- **Parts of the circle:**

A circle divides the plane on which it lies into three parts.

(*i*) inside the circle called interior.

(*ii*) outside the circle called exterior

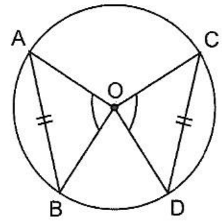
(*iii*) the circle



Angle Subtended by a Chord at a Point

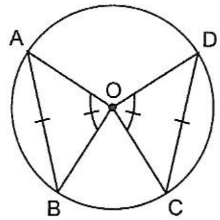
- Circles having the same centre are called concentric circles.
- Two circles are congruent if they have equal radii.

- **Theorem 1:** Equal chords of a circle subtend equal angles at the centre.



In the figure, if
AB = CD, then
∠AOB = ∠COD

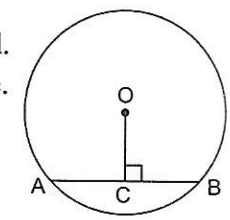
- **Theorem 2:** If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.



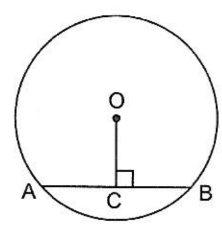
In the figure,
If ∠AOB = ∠COD, then
AB = CD

Perpendicular from the Centre to a Chord

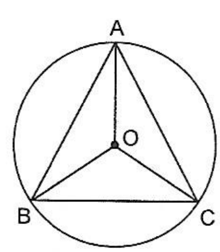
Theorem 3: The perpendicular drawn from the centre of a circle to a chord bisects the chord. In figure, 'O' is the centre of the circle, OC ⊥ AB where AB is a chord of the circle.
⇒ AC = BC



Theorem 4: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord. In the adjoining figure,
'O' is the centre of the circle and AC = BC
⇒ OC ⊥ AB



- **Circle through three points:**
  There is one and only one circle passing through three given non-collinear points.
- If ABC is a triangle, there is a unique circle passing through the three vertices A, B and C of the triangle. This circle is called the circumcircle of the ΔABC.
  Its centre and radius are called the circumcentre and circumradius of the triangle.
  In the adjoining figure, 'O' is circumcentre and OA, OB, OC are circumradius of circumcircle of ΔABC.

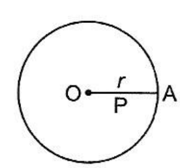


SOLVED QUESTIONS BASED ON EXERCISES 10.1, 10.2 AND 10.3

Very Short Answer Type Questions [1 Mark]

1. Given a circle of radius r and with centre O. A point P lies in the plane, such that OP < r. Tell the exact position of point P.

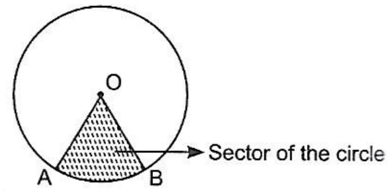
Sol. Let a point A lie on the circle as shown in the figure.



Then $OA = r$
and $OP < r$
⇒ $OP < OA$

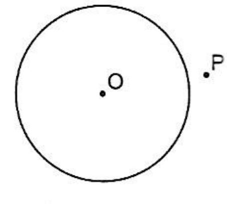
This shows point P lies inside the circle.

2. **Name the region between an arc and two radii, joining the centre to the end points of the arc.**



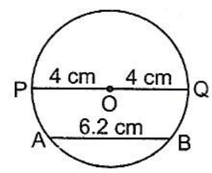
Sol. This region is called the sector of the circle as shown in the figure.

3. **In the given figure, tell the position of point P.**



Sol. Point P lies outside the circle.

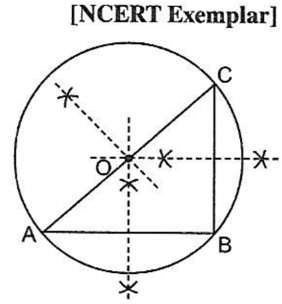
4. **Justify the statement: A circle of radius 4 cm can be drawn through two points A and B, such that AB = 6.2 cm.**



Sol. It is true that a circle of radius 4 cm can be passed through two points A and B, where AB = 6.2 cm.
If we draw a circle of radius 4 cm, the length of longest chord, i.e. diameter = 8 cm.
Such diameter > AB = 6.2 cm. Hence, a chord of 6.2 cm can be drawn in a circle as shown in the figure.

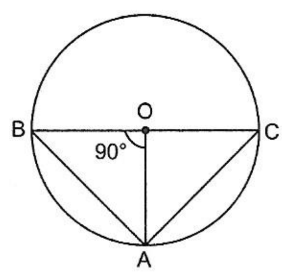
Short Answer Type Questions I [2 Marks]

5. **A, B and C are three points on a circle. Prove that perpendicular bisectors of AB, BC and CA are concurrent.** **[NCERT Exemplar]**



Sol. Given: A, B and C are three points on the circle.
To prove: Perpendicular bisector of AB, BC and CA are concurrent.
Proof: (*i*) Draw the perpendicular bisectors of AB.
(*ii*) Draw perpendicular bisector of BC. Both the perpendicular bisectors intersect at a point 'O'. This point 'O' is called the centre of the circle.
(*iii*) Now, draw perpendicular bisector of AC. We observe that perpendicular bisector of AC also passes through the same point O.
Hence, all the three perpendicular bisectors are concurrent, i.e. they pass through the same point.
(**Reason:** Three or more lines passing through the same point are called concurrent lines).

6. **In the given figure, AB = AC and O is the centre of the circle. If ∠BOA = 90°, determine ∠AOC.**



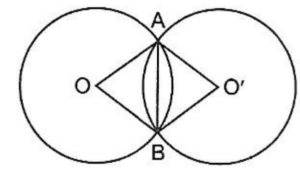
Sol. Given: A circle having centre O and AB = AC. Also, ∠AOB = 90°
To find ∠AOC
Proof: As we know that equal chords of a circle subtend equal angles at the centre of the circle
∴ ∠AOB = ∠AOC (as AB = AC)
⇒ ∠AOC = 90° (as ∠AOB = 90°)

Short Answer Type Questions II [3 Marks]

7. Two congruent circles with centres O and O′ intersect at two points A and B. Then ∠AOB = ∠AO′B. Write True or False, and justify your answer. **[NCERT Exemplar]**

Sol. **Given:** Two circles with centres O and O′ are congruent. AB is the common chord.

Then ∠AOB = ∠AO′B (True)



Construction: Join OA, OB, O′A and O′B.

Justification: In ΔAOB and ΔAO′B

OA = O′A (Radii of congruent circles)

OB = O′B (Radii of congruent circles)

AB = AB (common)

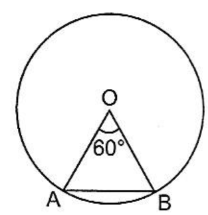
∴ ΔAOB ≅ ΔAO′B (By SSS congruence rule)

⇒ ∠AOB = ∠AO′B (By CPCT)

Hence proved. Therefore, it is true

8. In the given figure, chord AB subtends ∠AOB equal to 60°. at the centre O of the circle. If OA =5 cm. then find the length of AB.

Sol. **Given:** ∠AOB = 60°, OA = 5 cm, where O is the centre of the circle.



To find: AB

Proof: In ΔAOB

∠AOB = 60° (Given)

OA = OB (Equal radii)

∴ ∠OAB = ∠OBA (Angles opposite to equal sides OA and OB) ...(*i*)

In ΔAOB

∠OAB + ∠AOB + ∠OBA = 180° (Angle sum property of triangle)

60° + ∠OAB + ∠OAB = 180° (∵ ∠OAB = ∠OBA, using (*i*))

⇒ 2∠OAB = 180° – 60°

⇒ ∠OAB = 60°

⇒ ∠OBA = 60°

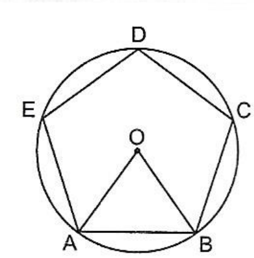
∴ ΔAOB is an equilateral triangle

Hence OA = OB = AB

⇒ AB = 5 cm (as OA = 5 cm)

Long Answer Type Questions [4 Marks]

9. In the given figure, O is the centre of a circle and A, B, C, D and E are points on the circle such that AB = BC = CD = DE = EA. Find the value of ∠AOB. **[CBSE 2016]**



Sol. **Given:** O is centre of circle and AB = BC = CD = DE = EA.

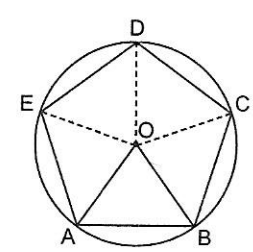
Construction: Join OC, OD, OE

To find: ∠AOB

Proof: A, B, C, D and E are the points which lie on the circle.

Also AB = BC = CD = DE = EA

All are the chords of the circle



As we know that equal chords subtend equal angle at the centre of circle.

∴ ∠AOB = ∠BOC = ∠COD = ∠DOE = ∠AOE ...(*i*)

Also, ∠AOB + ∠BOC + ∠COD + ∠DOE + ∠EOA = 360°

(sum of angles at the centre of circle)

Using (i)

$$\angle AOB + \angle AOB + \angle AOB + \angle AOB + \angle AOB = 360°$$

$\Rightarrow$ $5\angle AOB = 360°$

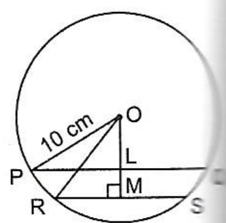
$\therefore$ $\angle AOB = 72°$

10. PQ and RS are two parallel chords of a circle on the same side of centre O and radius is 10 cm. I
PQ = 16 cm and RS = 12 cm, find the distance between the chords.

Sol. Given: A circle with centre O and two chords PQ and RS, such that PQ | | RS

To find: LM

Construction: Draw OM ⊥ RS which intersects PQ at L.



Proof: $\because$ OM ⊥ RS

$\therefore$ OL ⊥ PQ ($\because$ PQ | | RS)

$\therefore$ $PL = \frac{1}{2}PQ$ and $RM = \frac{1}{2}RS$

Now, PL = 8 cm and RM = 6 cm

Let LM = x cm

OP = OR = 10 cm

In ΔOPL, $OL = \sqrt{(10)^2 - (8)^2}$ cm = 6 cm

Also, In ΔORM, $OM = \sqrt{(10)^2 - (6)^2}$ cm = 8 cm

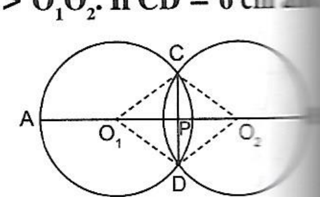
$\therefore$ $x = OM - OL = 8$ cm $- 6$ cm

$\Rightarrow$ $x = 2$ cm

$\therefore$ Distance between the chords = LM = 2 cm

11. O_1 and O_2 are the centres of two congruent circles intersecting each other at points C and D. The
joining their centres intersects the circles in points A and B such that AB > O_1O_2. If CD = 6 cm
AB = 12 cm, determine the radius of either circle.

Sol. Let radius of each circle = r cm



AB = 12 cm

$\therefore$ $O_1O_2 = 12 - 2r$

Now, CD is the common chord of the two circles and O_1O_2 is the line segment that joins the centres. [Radii of congruent

As we know that line joining the centres of two circles is perpendicular bisector of the common chord

$\therefore$ $O_1O_2 \perp CD$ and O_1O_2 bisects CD

$\therefore$ $CP = \frac{1}{2} \times CD = 3$ cm.

and $O_1P = \frac{1}{2}(O_1O_2) = \frac{1}{2}(12 - 2r) = (6 - r)$ cm

Now in right ΔCPO_1, $(O_1C)^2 = (O_1P)^2 + (PC)^2$

$\Rightarrow$ $r^2 = (6 - r)^2 + (3)^2$

$\Rightarrow$ $r^2 = 36 + r^2 - 12r + 9$

$\Rightarrow$ $12r = 45$

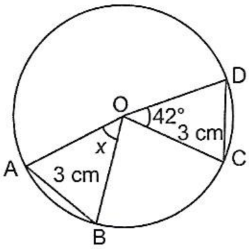
$\Rightarrow$ $r = \frac{45}{12}$

$\Rightarrow$ $r = 3.75$ cm

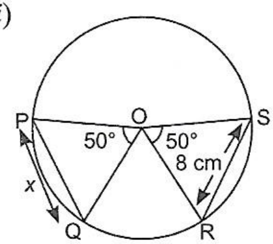
PRACTICE QUESTIONS BASED ON EXERCISES 10.1, 10.2 AND 10.3

1. Find the value of x in the following figures, where 'O' is the centre of the circle.

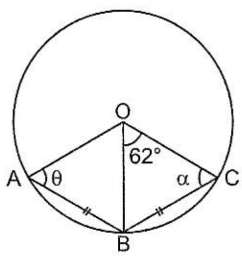
(*i*)



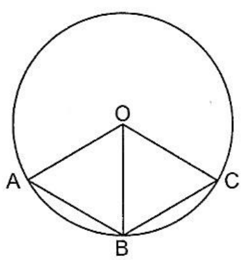
(*ii*)



2. Find the value of θ and α in the given figure, where 'O' is the centre of the circle.



3. In the given figure, OABC is rhombus. Find ∠AOB and ∠BOC, where 'O' is the centre of circle.

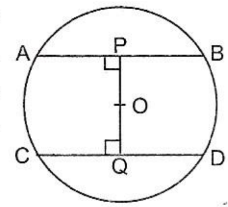


4. Distance of a chord AB of a circle from the centre is 12 cm and length of the chord AB is 10 cm. Find the diameter of the circle.

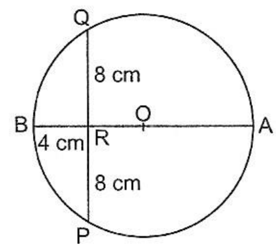
5. In a circle of radius 5 cm having centre O, OL is drawn perpendicular to the chord AB. If OL = 3 cm, find the length of AB.

6. In a circle with centre O, chords AB and CD are of lengths 4 cm each and subtend angles $x°$ and $y°$ at the centre of the circle respectively. Determine the relation between $x°$ and $y°$. **[CBSE 2011]**

7. In the given figure, O is the centre of the circle with radius 5 cm. OP ⊥ AB, OQ ⊥ CD, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.



8. In the given figure, diameter AB of circle with centre O bisects the chord PQ. If PR = QR = 8 cm and RB = 4 cm, find the radius of the circle.



9. A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B. Give reason to support your answer. **[HOTS]**

10. Prove that the line drawn through the centre of a circle to the mid point of a chord is perpendicular to the chord.

Equal Chords and Their Distances from the Centre

- The length of the perpendicular from a point to a line is the distance of the line from the point.
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres). Conversely, chords equidistant from the centre of a circle one equal in length.

SOLVED QUESTIONS BASED ON EXERCISE 10.4

Very Short Answer Type Questions [1 Mark]

1. **Justify your statement**

"The angles subtended by a chord at any two points of a circle are equal" **[NCERT Exemplar]**

Sol. The angles subtended by a chord at any two point of a circle are equal if both the points lie in the same segment (major or minor), otherwise they are not equal.

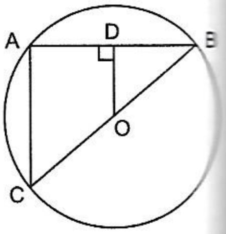
2. Justify your statement

"Two chords of a circle of lengths 10 cm and 8 cm are at the distances 8 cm and 3.5 cm respectively from the centre" **[NCERT Exemplar]**

Sol. The statement is not correct because the longer chord will be at smaller distance from the centre.

Short Answer Type Questions I [2 Marks]

3. If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle, show that CA = 2OD.



Sol. Given: A circle of centre O, diameter BC and OD⊥ chord AB.

To prove: CA = 2OD

Proof: Since OD ⊥ AB

∴ D is the mid-point of AB (perpendicular drawn from the centre to a chord bisects the chord)

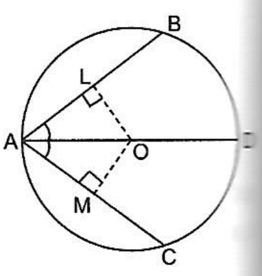
O is centre ⇒ O is the mid-point of BC.

In ΔABC, O and D are the mid-points of BC and AB, respectively.

∴ $OD \parallel AC$ and $OD = \frac{1}{2}AC$ (mid-point theorem)

∴ CA = 2OD

4. If two chords of a circle are equally inclined to the diameter passing through their point of intersection, prove that the chords are equal.



Sol. Given: Two chords AB and AC of a circle are equally inclined to diameter AOD, i.e.

$$\angle DAB = \angle DAC$$

Construction: Draw OL ⊥ AB and OM ⊥ AC

Proof: In ΔOLA and ΔOMA

$\angle OLA = \angle OMA$ (each 90°)

AO = AO (common)

$\angle OAL = \angle OAM$ (given)

ΔOLA ≅ ΔOMA (AAS rule)

⇒ OL = OM (CPCT)

⇒ AB = AC (chords equidistant from the centre are equal)

Short Answer Type Questions II [3 Marks]

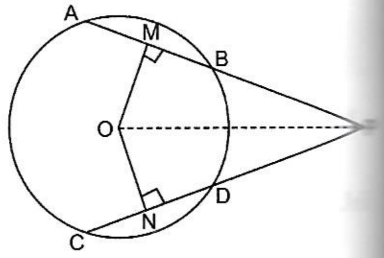
5. Two equal chords AB and CD of a circle when produced intersect at point P. Prove that PB = PD. **[NCERT Exemplar]**

Sol. Given: AB = CD, chords AB and CD when produced meet at point P.

To prove: PB = PD

Construction: draw OM ⊥ AB and ON ⊥ CD. Join OP where O is the centre of circle.

Proof: In ΔPOM and ΔPON



OM = ON (Equal chords of a circle are equidistant from the centre)

$\angle OMP = \angle ONP = 90°$ (by construction)

OP = OP (common)

∴ ΔOMP ≅ ΔONP (by RHS)

$\therefore$ PM = PN (by CPCT) ...(i)

As AB = CD (given)

$\frac{1}{2}AB = \frac{1}{2}CD$

BM = DN ...(ii) (perpendicular drawn from the centre on the chord bisects the chord)

Subtracting (ii) from (i) PM – BM = PN – DN

$\Rightarrow$ PB = PD

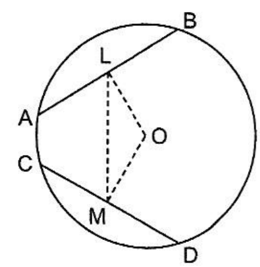
Long Answer Type Questions [4 Marks]

6. Prove that the line segment joining the mid-points of two equal chords of a circle make equal angles with the chords.

Sol. **Given:** A circle C(O, r), AB and CD are two equal chords of a circle. L, M are the mid-points of AB and CD respectively.

To Prove: (i) $\angle ALM = \angle CML$

(ii) $\angle BLM = \angle DML$.



Construction: LM, OL, OM are joined.

Proof : (i) $OL \perp AB$ and $OM \perp CD$.

(As the line joining the centre to the mid-point of the chord is perpendicular to the chord)

Now, OL = OM [Equal chords are equidistant from the centre]

In ΔOLM, OL = OM [Proved above]

$\Rightarrow$ $\angle OLM = \angle OML$ [Angles opposite to equal sides are equal] ...(i)

$\angle OLA = \angle OMC$ [Each 90°]

$\Rightarrow$ $\angle OLA - \angle OLM = \angle OMC - \angle OML$ [$\because \angle OLA = \angle OMC = 90°$]

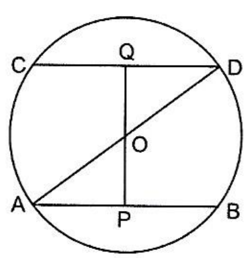
$\Rightarrow$ $\angle MLA = \angle LMC$ ...(ii)

Again from (i)

$\angle OLM + \angle OLB = \angle OML + \angle OMD$ [$\because \angle OLB = \angle OMD = 90°$]

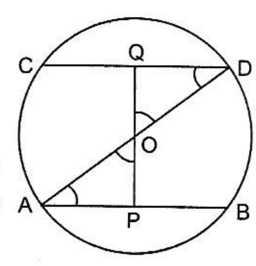
$\Rightarrow$ $\angle MLB = \angle LMD$.

7. In the given figure, AB || CD. AD is a diameter of the circle whose centre is O. Prove that AB = CD.



Sol. **Given:** AB || CD, AOD is the diameter of circle, where O is the centre of circle.

To prove: AB = CD



Proof: In ΔDOQ and ΔAOP

OD = OA (radii of circle)

$\angle DOQ = \angle AOP$ (vertically opposite angle

$\angle QDO = \angle PAO$ [alternate angles as CD || AB (given)]

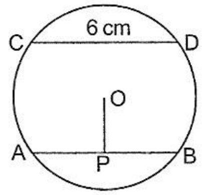
$\Rightarrow$ $\Delta DOQ \cong \Delta AOP$ (by ASA)

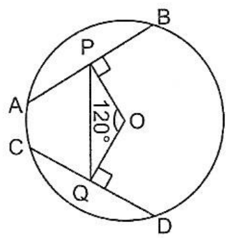
$\Rightarrow$ OQ = OP (by CPCT)

$\Rightarrow$ CD = AB (chords equidistant from the centre of a circle are equal)

PRACTICE QUESTIONS BASED ON EXERCISE 10.4

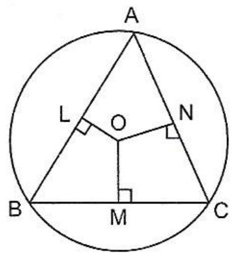
1. Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the angle bisector of $\angle BAC$.
2. In a circle of radius 18 cm, AB and AC are two chords such that AB = AC = 12 cm. Find the length of chord BC.
3. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre. Prove that $4q^2 = p^2 + 3r^2$.
4. In the given figure, AB and CD are two chords equidistant from the centre O. OP is the perpendicular drawn from centre O to AB. If CD = 6 cm, find PB. **[CBSE 2016]**



5. 

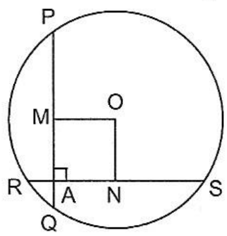
In the given figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If $\angle POQ = 120°$, find $\angle APQ$ **[CBSE 2014]**

6. In figure, O is the centre of the circle, OM $\perp$ BC, OL $\perp$ AB, ON $\perp$ AC and OM = ON = OL **[CBSE 2011]**



Prove that ΔABC is an equilateral triangle.

7. Two equal chords PQ and RS of a circle with centre O, intersect each other at right angle A. If M and N are mid-points of the chord PQ and RS respectively, then prove that OMAN is a square.



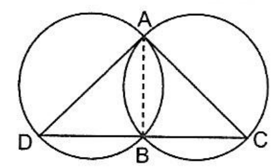
Angle Subtended by an Arc of a Circle and Cyclic Quadrilaterals

- If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.
- Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle arc equal. Conversely, if a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).
- Angle in a semicircle is a right angle.
- A quadrilateral is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180°. Conversely, if the sum of pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

SOLVED QUESTIONS BASED ON EXERCISE 10.5

Very Short Answer Type Questions [1 Mark]

1. Two circles intersect at two points A and B. AD and AC are diameters of the two circles. Prove that B lies on the line segment DC.



Sol. Given: Two circles intersect at A and B. AD and AC are diameters.

To prove: B lies on DC

Construction: Join AB

Proof: AD is the diameter of a circle.

$\therefore$ $\angle ABD = 90°$ ...(*i*) (Angle in a semicircle)

AC is the diameter of another circle.

$\therefore$ $\angle ABC = 90°$ ...(*ii*) (Angle in a semicircle)

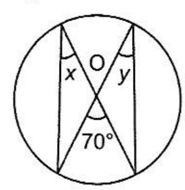
Adding (*i*) and (*ii*)

$$\angle ABD + \angle ABC = 90° + 90° = 180°$$

$\therefore$ These two angles from a linear pair angles

$\Rightarrow$ DBC is a line. Hence point B lies on line segment DC.

2. In the given figure, find the value of x and y where O is the centre of the circle.



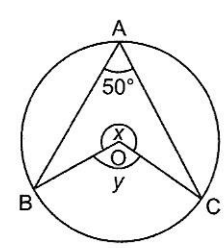
Sol. $y = \frac{1}{2} \times 70° = 35°$ (Angle at the centre is double the angle subtended by the same arc at any point on the remaining part of the circle)

Also $\angle x = \angle y$ (Angles in the same segment are equal)

$\Rightarrow$ $x = 35°$

Short Answer Type Questions I [2 Marks]

3. Find x in the adjoining figure.



Sol. Here O is the centre of the circle

$\therefore$ $\angle BAC = \frac{1}{2}\angle y$ (By degree measure theorem)

$\Rightarrow$ $50 = \frac{1}{2}\angle y$

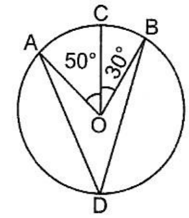
$\Rightarrow$ $\angle y = 100°$

Also $\angle x + \angle y = 360°$ (Angle at the centre of a circle)

$\Rightarrow$ $\angle x + 100° = 360°$

$\Rightarrow$ $\angle x = 360° - 100° = 260°$

4. In the given figure, O is the centre of the circle, $\angle AOC = 50°$ and $\angle COB = 30°$. Find the measure of $\angle ADB$. **[CBSE 2010]**



Sol. Here $\angle AOC = 50°$ and $\angle BOC = 30°$

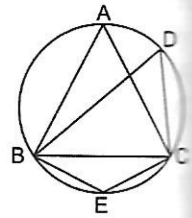
$\therefore$ $\angle AOB = \angle AOC + \angle BOC = 50° + 30° = 80°$

$\angle AOB = 80°$

$$\angle ADB = \frac{1}{2}\angle AOB \quad \text{(By degree measure theorem)}$$

$$\therefore \quad \angle ADB = \frac{1}{2} \times 80° = 40°$$

5. In the given figure, ΔABC is equilateral. Find ∠BDC and ∠BEC.



Sol. $\angle BAC = 60°$ [∵ ΔABC is an equilateral triangle]

$\therefore \quad \angle BAC = \angle BDC$ [∵ Angles in the same segment of a circle are equal]

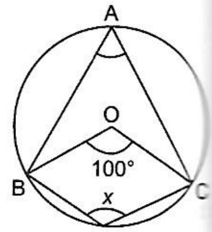
$\Rightarrow \quad \angle BDC = 60°$

Now, ☐ DBEC is a cyclic quadrilateral

$\therefore \quad \angle BDC + \angle BEC = 180°$ [∵ Opposite angles of a cyclic quadrilateral are supplementary]

$60° + \angle BEC = 180° \Rightarrow \angle BEC = 180° - 60° = 120°$

6. If ∠BOC = 100°, then find *x* from the given figure.



Sol. Here O is the centre of the circle

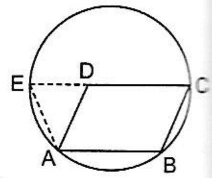
$\therefore \quad \angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 100° = 50°$

Also $\quad \angle x + \angle BAC = 180°$ (sum of opposite angles of cyclic quadrilateral)

$\Rightarrow \quad \angle x + 50° = 180° \Rightarrow x = 130°$

Short Answer Type Questions II [3 Marks]

7. ABCD is a parallelogram. The circles through A, B and C intersect CD (produced, if necessary) at E. Prove that AE = AD. **[NCERT]**



Sol. Given : ABCD is a parallelogram. A circle passes through A, B and C intersect side CD produced at E.

To Prove: $AE = AD$

Construction: Join AE

Proof: ABCD is a | |gm

$\therefore \quad \angle ADC = \angle ABC$ [Opposite angles of parallelogram] ...(*i*)

$\angle ADC + \angle ADE = 180°$ ...(*ii*) (Angles on straight line)

Also, $\quad \angle ABC + \angle AEC = 180°$ ...(*iii*) [(Angles of cyclic quadrilateral ABCE by construction]

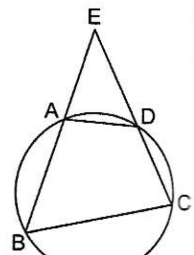
On equating (*ii*) and (*iii*)

$\angle ADC + \angle ADE = \angle ABC + \angle AEC$

$\Rightarrow \quad \angle ADE = \angle AEC$ (As ∠ADC = ∠ABC opposite angles of | |gm)

$\Rightarrow \quad AD = AE$ (Sides opposite to equal angles are equal)

8. ABCD is a cyclic quadrilateral. BA and CD produced meet at E. Prove that the triangles EBC and EDA are equiangular.



Sol. Given: ABCD is a cyclic quadrilateral. BA and CD are produced to meet at E.

To Prove: Δs EBC and EDA are equiangular.

Proof: ∵ ABCD is a cyclic quadrilateral.

$\therefore \quad \angle BAD + \angle BCD = 180°$ [Sum of opposite angles of a cyclic quadrilateral] ...(*i*)

But $\quad \angle BAD + \angle EAD = 180°$ [Linear pair] ...(*ii*)

From (*i*) and (*ii*),

$\angle BCD = \angle EAD$

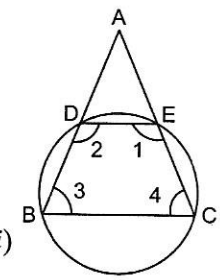
Similarly, $\quad \angle ABC = \angle EDA$

and $\quad \angle BEC = \angle AED$ [Common]

Hence, Δs EBC and EDA are equiangular.

9. ABC is an isosceles triangle in which AB = AC. A circle passing through B and C intersects AB and AC at D and E respectively. Prove that BC || DE.

Sol. Given: An isosceles triangle ABC, in which AB = AC and a circle through B and C intersecting AB and AC at D and E respectively.



To Prove: DE || BC.

Proof : In ΔABC, $AB = AC \Rightarrow \angle 3 = \angle 4$ [Angles opposite to equal sides are equal] ...(*i*)

Also, DBCE is a cyclic quadrilateral $\Rightarrow \angle 2 + \angle 4 = 180°$

[Opposite angles of a cyclic quadrilateral are supplementary]

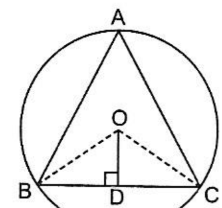
$\Rightarrow \quad \angle 2 + \angle 3 = 180°$ [From (*i*)] ...(*ii*)

But $\angle 2$ and $\angle 3$ are co-interior angles on the same side of transversal BD.

$\therefore$ DE || BC.

10. O is the circumcentre of the triangle ABC and OD is perpendicular to BC. Prove that $\angle BOD = \angle A$.

Sol. Given: In triangle ABC, $OD \perp BC$ where O is the circumcentre of ΔABC.



To prove: $\angle BOD = \angle A$

Construction: Join OB and OC

Proof: Here O is the centre of circle.

$\therefore \quad \angle BOC = 2\angle A$ ... (*i*) (By degree measure theorem)

Also, in ΔBOD and ΔCOD

$OB = OC$ (radii of circle)

$OD = OD$ (common)

$\angle ODB = \angle ODC = 90°$ ($OD \perp BC$ given)

$\Rightarrow \quad \Delta OBD \cong \Delta OCD$ (by RHS)

$\Rightarrow \quad \angle BOD = \angle COD$ (By CPCT) ...(*ii*)

$\Rightarrow \quad \angle BOC = \angle BOD + \angle COD$

$= \angle BOD + \angle BOD$ [Using (*ii*)]

$\Rightarrow \quad \angle BOC = 2\angle BOD$ ...(*iii*)

Equating (*i*) and (*iii*)

$2\angle A = 2\angle BOD$

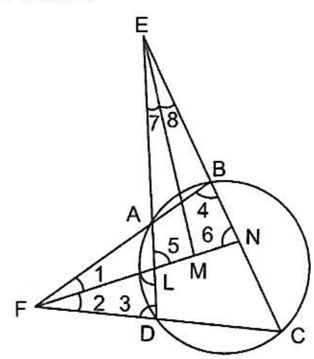
$\Rightarrow \quad \angle BOD = \angle A$

Long Answer Type Questions [4 Marks]

11. Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral (provided they are not parallel) intersect at right angle.

Sol. Given: ABCD is a cyclic quadrilateral whose opposite sides are produced to meet at E and F.

To Prove: Bisectors of $\angle E$ and $\angle F$ intersect at right angle.



Proof: In ΔFDL and ΔFBN,

$\angle 2 = \angle 1$ [$\because$ FN is the bisector of $\angle F$]

$\angle 3 = \angle 4$

[Exterior angle of cyclic quadrilateral is equal to interior opposite angle]

$\therefore$ Third $\angle FLD$ = Third $\angle 6$

But $\angle FLD = \angle 5$ [Vertically opposite angles]

$\therefore \quad \angle 5 = \angle 6$

$\Rightarrow \quad EN = EL$ [Sides opposite to equal angles are equal]

Now in ΔELM and ΔENM

$EL = EN$ (Proved above)

$EM = EM$ (Common)

$\angle 7 = \angle 8$ (Given as EM is the bisector of $\angle E$)

$\therefore$ $\Delta ELM \cong \Delta ENM$ (SAS congruence rule)

$\therefore$ $\angle EML = \angle EMN$ (CPCT)

But $\angle EML + \angle EMN = 180°$ (Linear pair)

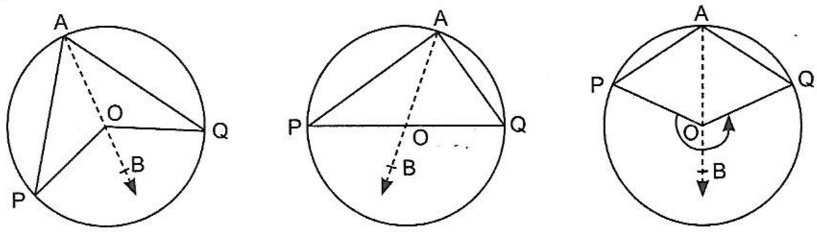
$\Rightarrow$ $\angle EML = \angle EMN = 90°$

Hence, $EM \perp FM$.

Hence, bisectors of $\angle E$ and $\angle F$ intersect at right angle.

12. Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Sol. Given : Given an arc PQ of a circle subtending angles POQ at the centre O and $\angle PAQ$ at a point A on the remaining part of the circle.



case (*i*) case (*ii*) case (*iii*)

To Prove: $\angle POQ = 2\angle PAQ$

Construction: Join AO and extends it to B.

Proof: Consider three cases

case (*i*): When arc PQ is a minor arc.

case (*ii*): When arc PQ is a semicircle.

case (*iii*): When arc PQ is a major arc.

In all the three cases

Taking ΔAOQ

$\angle BOQ = \angle OAQ + \angle OQA$ (Exterior angle of Δ is equal to the sum of interior opposite angles)

Also $OA = OQ$ (radii of circle)

$\Rightarrow$ $\angle OAQ = \angle OQA$ (Angles opposite to equal sides)

$\Rightarrow$ $\angle BOQ = \angle OAQ + \angle OAQ$

$\Rightarrow$ $\angle BOQ = 2\angle OAQ$ ...(*i*)

Similarly $\angle BOP = 2\angle OAP$ ...(*ii*)

Adding (*i*) and (*ii*) we have

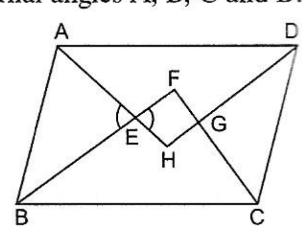
$$\angle BOQ + \angle BOP = 2\angle OAQ + 2\angle OAP$$
$$= 2(\angle OAQ + \angle OAP)$$

$\Rightarrow$ $\angle POQ = 2\angle PAQ$

Specially for case (iii) we can write reflex $\angle POQ = 2\angle PAQ$

13. Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic. **[NCERT]**

Sol. Given: ABCD is a quadrilateral, AH, BF, CF and DH are the angle bisectors of internal angles A, B, C and D. These bisectors form a quadrilateral EFGH.



To Prove: EFGH is cyclic

Proof: In ΔAEB.

$\angle EAB + \angle ABE + \angle AEB = 180°$ (Sum of angles of ΔABC)

$\Rightarrow$ $\angle AEB = 180° - (\angle EAB + \angle ABE)$ ...(*i*)

Also $\angle AEB = \angle FEH$ ...(*ii*) (Vertically opposite angles)

By equating (*i*) and (*ii*) $\angle FEH = 180° - (\angle EAB + \angle ABE)$ ...(*iii*)

Similarly, in ΔGDC $\angle FGH = 180° - (\angle GDC + \angle GCD)$ ...(*iv*)

Adding (*iii*) and (*iv*)

$$\angle FEH + \angle FGH = 360° - (\angle EAB + \angle ABE + \angle GDC + \angle GCD)$$

$$= 360° - \frac{1}{2}(\angle BAD + \angle ABC + \angle ADC + \angle BCD)$$

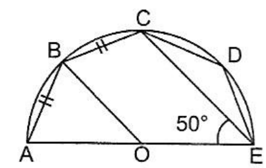
(As AH, BF, CF and HD are bisectors of $\angle A$, $\angle B$, $\angle C$, $\angle D$)

$$= 360° - \frac{1}{2} \times 360° \quad \text{(Sum of angles of quadrilateral, ABCD)}$$

$$\angle FEH + \angle FGH = 360° - 180° = 180°$$

$\Rightarrow$ FEHG is a cyclic quadrilateral. (If the sum of opposite angles of quadrilateral is 180°, then it is cyclic)

14. In the given figure, O is the centre and AE is the diameter of the semicircle ABCDE. If AB = BC and $\angle AEC = 50°$, then find (*a*) $\angle CBE$ (*b*) $\angle CDE$ (*c*) $\angle AOB$. Prove that BO || CE.



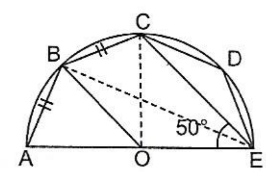
Sol. Given: O is the centre of circle and AE is the diameter of the semicircle ABCDE.

Also, AB = BC, $\angle AEC = 50°$

To find: (*a*) $\angle CBE$ (*b*) $\angle CDE$ (*c*) $\angle AOB$

and prove that BO || CE

Construction: Join OC and BE



Proof: $\angle AOC = 2\angle AEC$ (By degree measure theorem)

$\angle AOC = 2 \times 50° = 100°$

Also $\angle AOB = \angle BOC$ (Equal chords subtend equal angle at the centre of circle)

$\therefore$ $\angle AOB = \frac{1}{2}\angle AOC$ [Using (*i*)]

$\angle AOB = \frac{1}{2}(100°) = 50°$

Now $\angle AOB = \angle AEC$ (These are corresponding angles)

But these are corresponding angles and are equal.

$\therefore$ Line OB || CE Hence proved

(*a*) $\angle AOC + \angle COE = 180°$ (Linear pair angles)

$100° + \angle COE = 180°$

$\angle COE = 180° - 100° = 80°$

$\angle CBE = \frac{1}{2}\angle COE$ (By degree measure theorem)

$= \frac{1}{2} \times 80° = 40°$

(*b*) Now, □CBED is a cyclic quadrilateral.

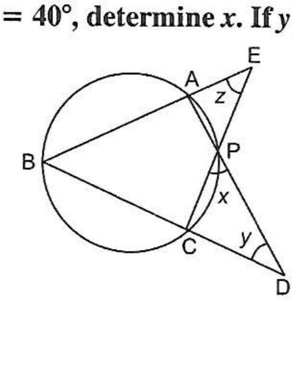
$\angle CBE + \angle CDE = 180°$ (Sum of opposite angles of cyclic quadrilateral)

$\Rightarrow$ $40° + \angle CDE = 180°$

$\Rightarrow$ $\angle CDE = 180° - 40° = 140°$

(*c*) $\angle AOB = 50°$ (Proved above)

15. In the given figure, if $y = 32°$ and $z = 40°$, determine x. If $y + z = 90°$, prove that $x = 45°$.



Sol. **Given:** $y = 32°$ and $z = 40°$.

Proof: Let the line segments AD and CE cut each other at P.

Since, $\angle APE = \angle CPD$ [Vertically opposite angles]

$\therefore$ $\angle APE = x$

Now, $\angle BCP = \angle CDP + \angle CPD$ [Exterior angle]

and $\angle PAB = \angle PEA + \angle APE$ [Exterior angle]

$\therefore$ $\angle BCP = x + y$ ... (*i*)

and $\angle PAB = x + z$ ... (*ii*)

Since, ABCP is a cyclic quadrilateral

$\therefore$ $\angle BCP + \angle PAB = 180°$

$\Rightarrow$ $x + y + x + z = 180°$

or $2x + (y + z) = 180°$ ... (*iii*)

or $2x + (40° + 32°) = 180°$

or $2x = 180° - 72° = 108°$ or $x = 54°$

Since, from (*iii*), we get

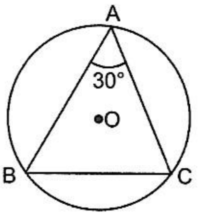
$2x + (y + z) = 180°$ and $y + z = 90°$ [Given]

$\therefore$ $2x + 90° = 180°$ or $2x = 90°$

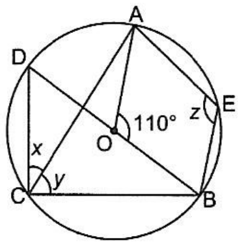
$\therefore$ $x = 45°$.

PRACTICE QUESTIONS BASED ON EXERCISE 10.5

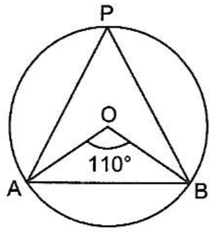
1. In the given figure, ABC is a triangle in which $\angle BAC = 30°$. Show that BC is equal to the radius of the circumcircle of ΔABC, whose centre is O. **[CBSE 2015]**



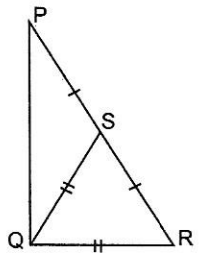
2. In the given figure, O is the centre of the circle and $\angle AOB = 110°$, find the value of x, y and z. **[CBSE 2015]**



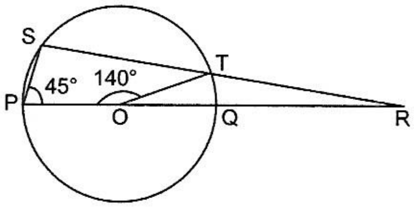
3. In the given figure, O is the centre of the circle and AB is a chord of the circle. If $\angle AOB = 110°$, find $\angle APB$.



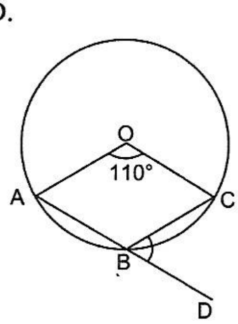
4. In ΔPQR, right-angled at Q. A point S is taken on the side PR such that PS = SR and QR = QS. Find the measure of $\angle QRS$. **[HOTS]**



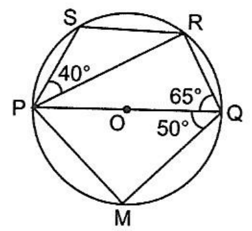
5. If O is centre of circle as shown in figure, find $\angle RQT$ and $\angle RTQ$



6. If O is the centre of the circle as shown in figure, find $\angle CBD$.

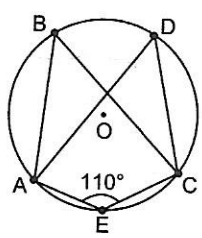


7. In the given figure, PQ is a diameter of a circle with centre O. If ∠PQR = 65°, ∠SPR = 40°, ∠PQM = 50°, find ∠QPR, ∠PRS and ∠QPM.

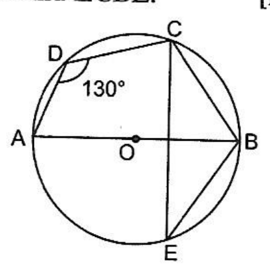


8. In the given figure, ABCE is a cyclic quadrilateral and O is the centre of circle. If ∠AEC = 110°, then find

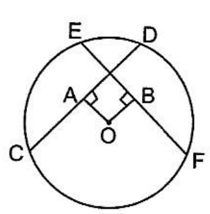
(*a*) ∠ABC (*b*) ∠ADC **[CBSE 2010]**



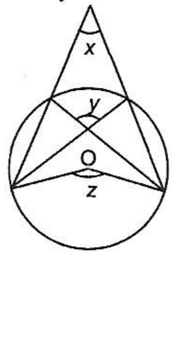
9. In the given figure, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE. **[NCERT Exemplar]**



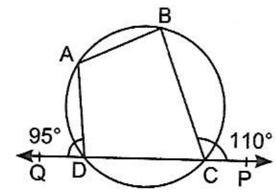
10. In the given figure, OA and OB are respectively perpendiculars to chords CD and EF of a circle whose centre is O. If OA = OB, prove that $\overset{\frown}{EC} = \overset{\frown}{DF}$.



11. In the given figure, O is the centre of the circle. Prove that ∠*x* + ∠*y* = ∠*z*

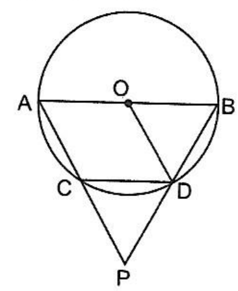


12. In the given figure, ABCD is a cyclic quadrilateral. Side CD is produced on both sides, such that ∠BCP = 110° and ∠ADQ = 95°. Find the values of ∠A and ∠B.

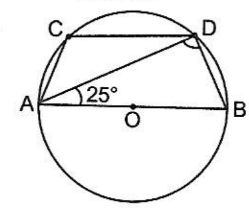


13. ABCD is a quadrilateral inscribed in a circle. CD is produced to any point F and the bisector of ∠ABC intersects the circle at E. Prove that DE bisects ∠ADF.

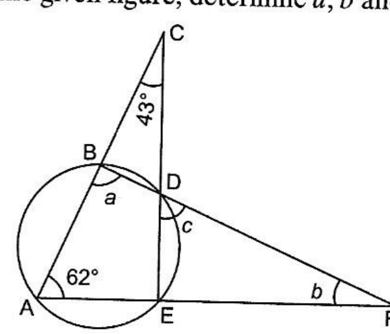
14. AB is a diameter of circle C(O, *r*). Chord CD is equal to radius OD. AC and BD produced intersect at P. Prove that ∠APB = 60°.



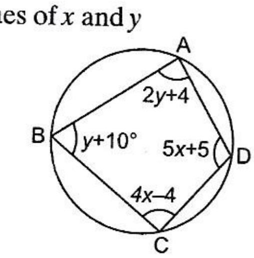
15. In the given figure, AB is diameter of the circle with centre O and CD || AB. If ∠DAB = 25°, then find the measure of ∠CAD.



16. In the given figure, determine *a*, *b* and *c*.

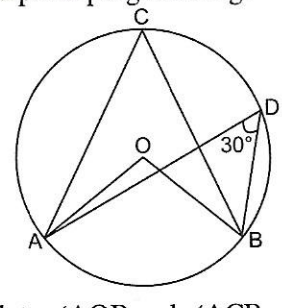


17. Find values of *x* and *y*



Value Based Questions

1. A non-government organisation has a circular piece of land as shown. It wants to develop a vocational training institute for poor girls to provide tailoring skills and computer programming.



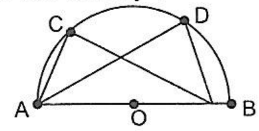
   (i) Calculate ∠AOB and ∠ACB, where O is the centre of circular land.

   (ii) What impression did you get through this activity?

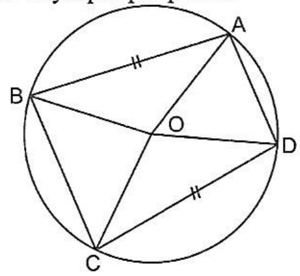
2. A non-permanent shade like structure is formed on road- side of semicircular type to provide guidelines of road safety. Also, O is the centre of semicircular shade:

   (i) Calculate ∠ADB and ∠ACB.

   (ii) What impression does the society receive through this activity ?



3. A sports academy has developed circular region (North-east region mainly naxal dominated) as shown to create various sports culture and to train rural boys for Olympic purposes.



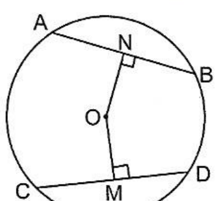
   (i) Find the relation between ∠AOB and ∠COD, where O is the centre of circle and both AB and CD are equal.

   (ii) What conclusion is derived from this activity?

INTEGRATED EXERCISE

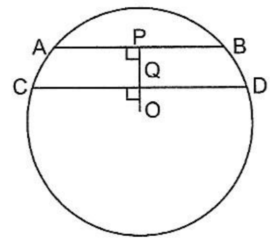
Very Short Answer Type Questions [1 Mark]

1. The radius of a circle is 10 cm and a chord of a circle is 12 cm in length. Find the distance of the chord from the centre of the circle.
2. Given a circle with centre O, having chords AB, PQ and XY. If points P, Q and O are collinear and radius of circle is 6 cm, find PQ.
3. Given a circle with centre O and the smallest chord AB = 3 cm, largest chord CD = 10 cm and chord PQ = 7 cm. Find the radius of the circle.
4. Find the length of a chord which is at a distance of 4 cm from the centre of a circle whose radius is 5 cm.
5. In the given figure, O in the centre of the circle. If OM < ON, show that CD > AB.

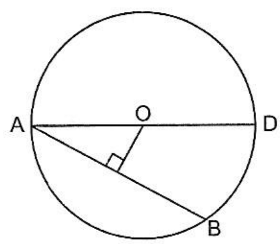


Short Answer Type Questions I [2 Marks]

6. In the given figure, O is the centre of the circle of radius 5 cm. OP ⊥ AB, OQ ⊥ CD, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.



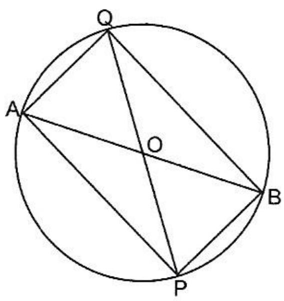
7. AD is a diameter of a circle and AB is a chord. If AD = 34 cm and AB = 30 cm, find the distance of AB from the centre of the circle.



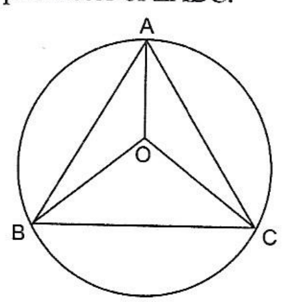
8. Prove that the diameter of a circle that bisect a chord also bisects the angle subtended by the chord at the centre of the circle. **[HOTS, CBSE 2016]**
9. Prove that the circle drawn on any equal side of an isosceles triangle as diameter bisects the base.
10. In a cyclic quadrilateral PQRS, if ∠P – ∠R = 50°, find ∠P. **[HOTS]**

Short Answer Type Questions II [3 Marks]

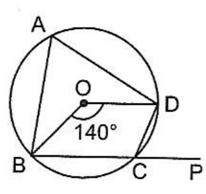
11. In the given figure, a diameter AB of a circle bisects a chord PQ and AQ || BP. Prove that chord PQ is also a diameter. What is the name given to quadrilateral AQBP. **[CBSE 2014]**



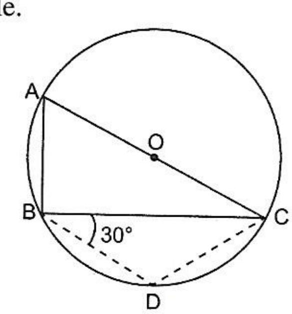
12. In the given figure, O is the centre of the circle, $\angle AOB = \angle BOC = \angle COA$ and $AO = 2\sqrt{3}$ cm. Find the perimeter of ΔABC. **[CBSE 2016]**



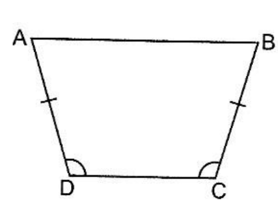
13. In the given figure, O is the centre of the circle. The angle subtended by arc BCD at the centre is 140°. BC is produced to P. Determine $\angle BAD$ and $\angle DCP$.



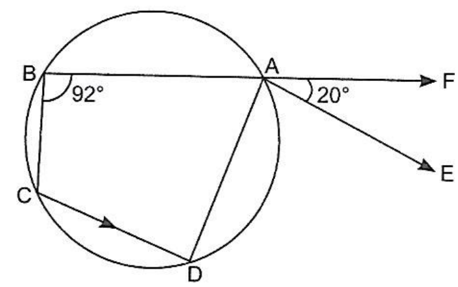
14. In the given figure, BD = DC and $\angle DBC = 30°$. Find the measure of $\angle BAC$, if O is the centre of the circle.



15. In the given figure, ABCD is a quadrilateral in which AD = BC and $\angle ADC = \angle BCD$. Show that the points A, B, C and D lie on a circle.

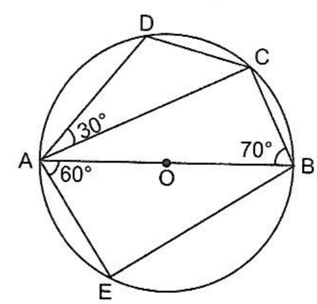


16. In the given figure, ABCD is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced up to F. If $\angle ABC = 92°$, $\angle FAE = 20°$, find $\angle BCD$.

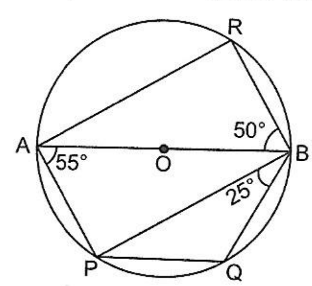


17. Two circles are drawn with sides AB and AC of a triangle ABC as diameters. The circles intersect at a point D. If AB = 5 cm, BD = 3 cm and AC = 6 cm, find BC.

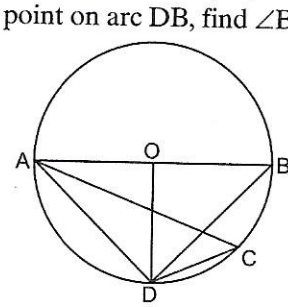
18. In the given figure, AB is a diameter of a circle with centre O. If $\angle ABC = 70°$, $\angle CAD = 30°$ and $\angle BAE = 60°$, find $\angle BAC$, $\angle ACD$ and $\angle ABE$.



19. In the given figure , AB is a diameter of a circle with centre O. If $\angle PAB = 55°$, $\angle PBQ = 25°$ and $\angle ABR = 50°$, find $\angle PBA$, $\angle BPQ$ and $\angle BAR$.

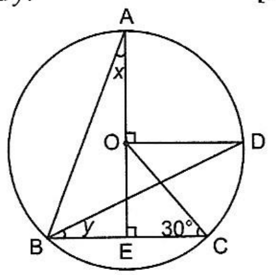


20. In the given figure, AB is a diameter of circle C(O, r) and radius OD is perpendicular to AB. If C is any point on arc DB, find $\angle BAD$ and $\angle ACD$.



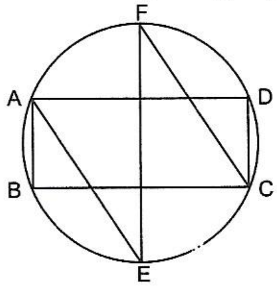
Long Answer Type Questions [4 Marks]

21. In figure O is the centre of the circle, $\angle BCO = 30°$. Find x and y. **[NCERT Exemplar]**

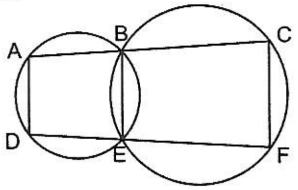


22. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic. **[NCERT Exemplar]**

23. The bisectors of the opposite angles A and C of a cyclic quadrilateral ABCD intersect the circle at the points E and F, respectively. Prove that EF is a diameter of the circle (See figure).

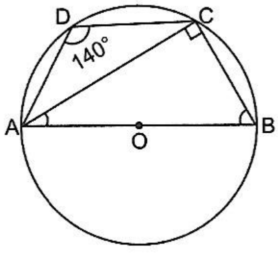


24. In the given figure, B and E are points on line segment AC and DF respectively. Show that AD || CF.

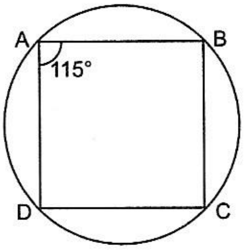


25. ABCD is a rectangle. The line through C perpendicular to AC meets AB produced at X and AD produced at Y. Prove that the points D, B, X, Y are concyclic.

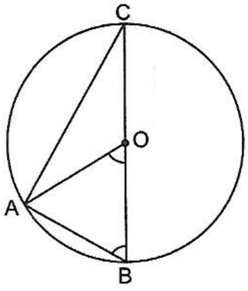
26. (*i*) In the given figure, ABCD is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B, C and D. If $\angle ADC = 140°$, find $\angle BAC$.



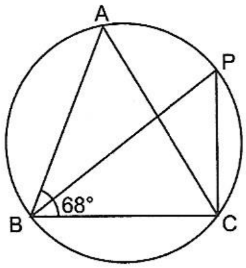
(*ii*) In the given figure, AB || DC. Determine $\angle DCB$ and $\angle ABC$.



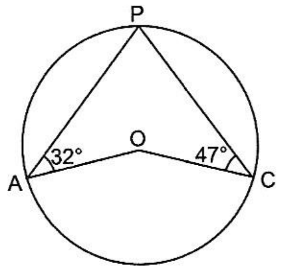
27. (*i*) In figure, O is the centre of the circle and $\angle AOB = 70°$. Find $\angle OBA$ and $\angle OAC$.



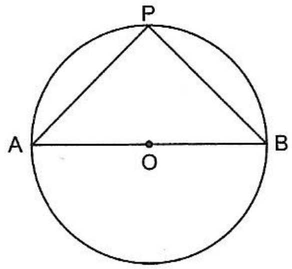
(*ii*) In the given figure, AB = AC and $\angle ABC = 68°$. Determine $\angle BPC$.



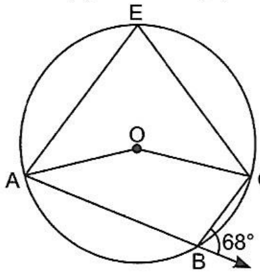
28. (*i*) In the given figure, O is the centre of the circle. Calculate $\angle APC$ and $\angle AOC$.



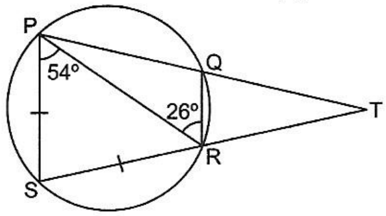
(*ii*) In the given figure, O is the centre of the circle and AP = BP. Calculate $\angle A$ and $\angle POA$.



29. (*i*) In the given figure, O is the centre of the circle. Determine (*a*) ∠AEC (*b*) Reflex ∠AOC.

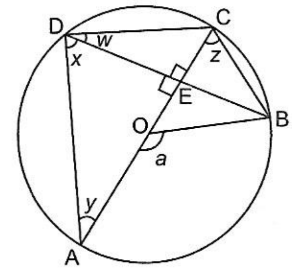


(*ii*) In the given figure, PS = SR, ∠RPS = 54° and ∠PRQ = 26°. Calculate (*a*) ∠TQR (*b*) ∠RTQ.



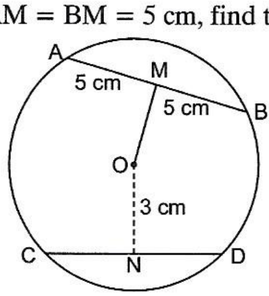
30. ABC is a triangle inscribed in a circle with centre O. If ∠AOC = 130° and ∠BOC = 150°, find ∠ACB.

31. In the given figure, AC is the diameter of circle with centre O. Chord BD is perpendicular to AC. Find measures of angles *x*, *y*, *z*, *w* in terms of *a*.

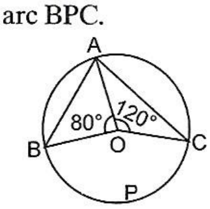


ASSESS YOURSELF

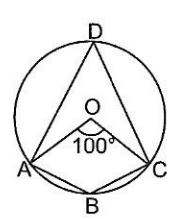
1. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at point of minor arc.
2. Two congruent circles have centres O_1 and O_2. Arc AB of a circle, with centre O, subtends an angle of 100° at the centre and an arc A′B′ of the circle, with centre O_2, subtends an angle of 25°. Find arc (AB): arc (A′B′) **[HOTS]**
3. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.
4. In the given figure, O is the centre of the circle. AB and CD are two chords of the circle. Also, OM ⊥ AB and ON ⊥ CD. If OM = ON = 3 cm and AM = BM = 5 cm, find the chord CD.



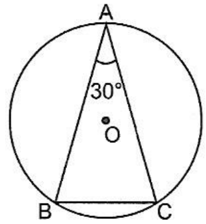
5. AB and CD are two parallel chords of a circle whose centre is O and radius is 20 cm. If AB = 24 cm and CD = 32 cm, find the distance between AB and CD, if its given that they lie.
   (*i*) on opposite sides of centre O.
   (*ii*) on the same side of centre O. **[CBSE, 2015]**
6. Prove that an isosceles trapezium is always cyclic and its diagonal are equal. **[CBSE 2016]**
7. In the given figure, A, B, C are three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 80° and 120° respectively. Determine ∠BAC and the degree measure of arc BPC.



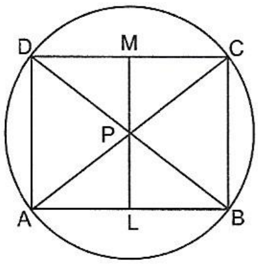
8. In the given figure, O is the centre of the circle and the measure of arc ABC is 100°. Determine ∠ADC and ∠ABC.



9. In the given figure, ABC is a triangle in which ∠BAC = 30°. Show that the length of BC is equal to the radius of the circumcircle of ΔABC, whose centre is O.



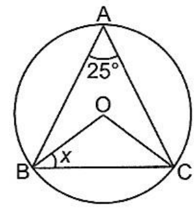
10. In the given figure, ∠BPC = 90°, LM ⊥ CD. If AB is 10 cm, find BL.



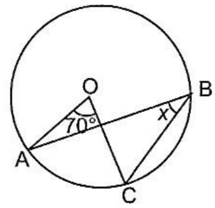
11. ABCD is a parallelogram. The circle through A, B and C intersects CD produced at E. If AB = 10 cm, BC = 8 cm, CE = 14 cm, find AE.

12. Prove that the angle subtended by an arc at the centre of a circle is double the angle subtended by it on any point on the remaining part of the circle. Use above result to find the following:

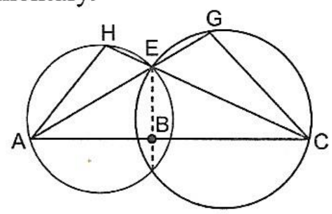
   (*i*) O is the centre of the circle and ∠BAC = 25°, find ∠x.



   (*ii*) Find x in the figure where O is the centre of the circle.



13. In the given figure, ABC, AEG and HEC are straight lines. Prove that ∠AHE and ∠EGC are supplementary.



14. The bisectors of the opposite angles P and R of a cyclic quadrilateral PQRS intersect the circle at the points A and B respectively. Prove that AB is a diameter of the circle.

15. In a triangle ABC, ∠A = 60° and the bisectors of ∠B and ∠C meet AC and AB at P and Q respectively and intersect each other at I. Prove that ∠BIC = 120° and APIQ is a cyclic quadrilateral.